Spreading in two-dimensional disordered nonlinear lattices

Final report for the HPC-Europa2 project number 666

Mario Mulansky

December 20, 2011

During my HPC-Europa2 project (Project Number 666), I performed numerical simulations for two different two-dimensional models: the 2D DANSE model and a 2D lattice of Hamiltonian oscillators. For both cases I was interested in the spreading behavior of initially localized modes. Results in one dimension have been obtained previously for both the DANSE model [1, 2, 3] and the Hamiltonian lattice [4]. Here, I report some of the major results obtained for the 2D version of the DANSE model. Due to limitations on the length of this report I am not able not show the results of the second model. However, they are currently being prepared for publication and should be available in the near future.

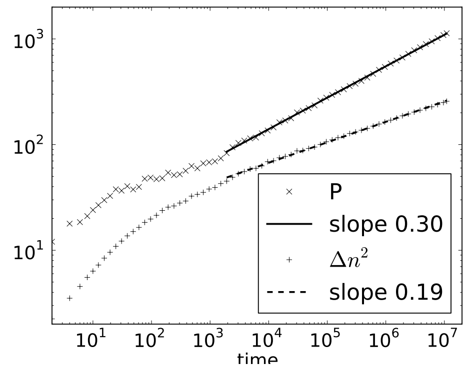
We are interested in the spreading of initially localized wave packets in a two dimensional disordered nonlinear Schrödinger lattice (2D DANSE model). A rather superficial investigation of this model has been performed previously [5] and our aim was to obtain more reliable results by using the CINECA sp6 supercomputer to reach longer time scales and perform better averaging. The equations of motions for the 2D DANSE are:

$$i\frac{\mathrm{d}}{\mathrm{d}t}\psi_{n,k} = V_{n,k}\psi_{n,k} + \psi_{n-1,k} + \psi_{n+1,k} + \psi_{n,k-1} + \psi_{n,k+1} + \beta|\psi_{n,k}|^2\psi_{n,k}. \qquad (1)$$

$\psi_{n,k}$ is the complex wave function amplitude at lattice site (n,k), $V_{n,k}$ is the random potential at this site chosen iid. from $[-W/2, W/2]$. The lattice was chosen quadratic with $N \times N$ sites and $N = 256$. β represents the nonlinearity strength and was set to $\beta = 1.0$ in our simulations. Note that norm and energy are conserved quantities in this system.

For the numerical simulations, we started with single site excitions at the lattice center with the potential value $V_{N/2,N/2}$ at this site chosen such that the energy equals zero while the norm of the wave function was always 1. The we used a 6th order accurate composition scheme based on a two-dimensional version of a multi-symplectic method [6] to numerically perform the time evolution of this localized initial state. The advantage of this multi-symplectic method is that it is explicit in space and implicit only in time which means that it does not require to solve a linear set of equations for each time step but only to find the root of a quadratic equation for each lattice site. This results in a better performance than classical symplectic schemes like Crank-Nicolson. The time step was fixed to $\Delta t = 0.2$ and we integrated up to $t = 10^7$. As result of the multi-symplectic method norm and energy were conseved with accuracy 10^{-3} during the whole integration.

These time evolutions were repeated for up to $M = 20$ disorder realization and the results presented below are averages over these disorder realizations. To quantify spreading we used



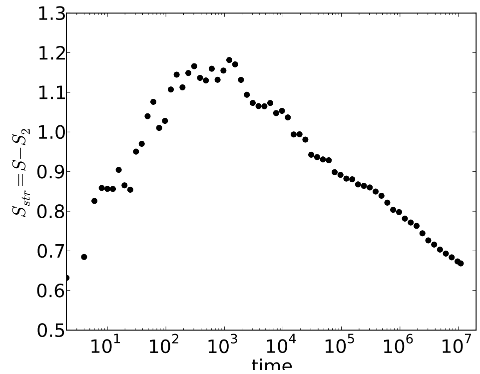


Figure 1: Left: Participation Number P and second moment Δn^2 vs. time for $W = 10$ averaged over 20 disorder realization. Right: Structural entropy $S_{\mathrm{str}} = S - S_2$ vs. time.

the well known quantity participation number P defined via $P^{-1} = \sum_{n,k} |\psi_{n,k}|^4$. P is understood to measures the number of excited sites in the system. Additionally, we used the second moment in x- and y-direction to obtain estimates of the extent of the excitation:

$$\Delta n^2 = \sqrt{\Delta x^2 \Delta y^2} \qquad \text{with} \begin{cases} \Delta x^2 = \sum_{n,k} n^2 |\psi_{n,k}|^2 - \bar{x}^2 & \text{and} \qquad \bar{x} = \sum_{n,k} n|\psi_{n,k}|^2 \\ \Delta y^2 = \sum_{n,k} k^2 |\psi_{n,k}|^2 - \bar{y}^2 & \text{and} \qquad \bar{y} = \sum_{n,k} k|\psi_{n,k}|^2. \end{cases} \tag{2}$$

The results for both measures are shown in Fig. 1 and Fig. 2 for two different values of disorder strength $W = 10$ and $W = 15$. In both cases we find $P \sim t^{0.3}$ while $\Delta n^2 \sim t^{0.2}$ which indicates that the structural properties of the states change during the spreading. This is fortified by the results on the structural entropy $S_{\mathrm{str}} := S - S_2$ [7] where S is the usual Shannon entropy and S_2 is the Rény entropy with index 2: $S_2 = -\ln \sum_{i,j} |\psi_{i,j}|^4 = \ln P$. If the spreading would be structurally self-similar we would observe that S_{str} is constant. The decreasing of S_{str} for $t > 10^3$ means the the wave function gets more and more uniform during the spreading progress. This change of the peak structure indicates that the asymptotic regime of self-similar spreading has not yet been reached in our simulations. However, we clearly observed a subdiffusive spreading as assumed from various results in one dimension and a previous study on this 2D setup [5].

Additionally, we studied this model with a different nonlinear term: $\beta|\psi_{n,k}|^{2/3}\psi_{n,k}$. For this setup a prediction exists saying that the spreading should be more efficient than the subdiffusive [8] behavior. Our results, however, showed usual subdiffusion and we could not identify any signature of enhanced spreading. Further analysis of these results are to be done.

The work has been performed under the HPC-EUROPA2 project (project number: 228398) with the support of the European Commission - Capacities Area - Research Infrastructures. M.M. thanks S. Lepri, A. Politi and the CNR in Florence, Italy for hospitality and financial support.

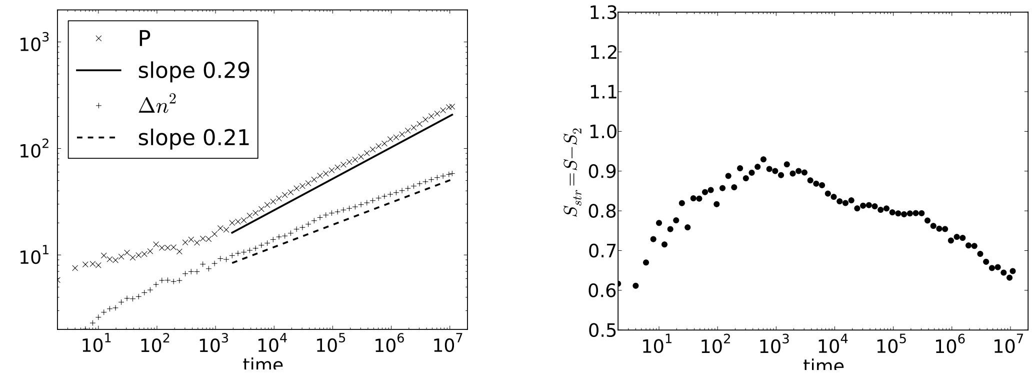


Figure 2: Same plot as in Fig. 1 but for disorder strength $W = 15$.

References

[1] A. S. Pikovsky and D. L. Shepelyansky. Destruction of Anderson localization by a weak nonlinearity. *Physical Review Letters*, 100(9):094101, 2008.

[2] M. Mulansky and A. Pikovsky. Spreading in disordered lattices with different nonlinearities. *Europhys. Lett.*, 90:10015, 2010.

[3] S. Flach, D. O. Krimer, and Ch. Skokos. Universal spreading of wave packets in disordered nonlinear systems. *Phys. Rev. Lett.*, 102(2):024101, 2009.

[4] M. Mulansky, K. Ahnert, and A. Pikovsky. Scaling of energy spreading in strongly nonlinear disordered lattices. *Phys. Rev. E*, 83:026205, 2011.

[5] Ignacio García-Mata and Dima L. Shepelyansky. Delocalization induced by nonlinearity in systems with disorder. *Physical Review E*, 79(2):026205, February 2009.

[6] Yu-Shun Wang, Qing-Hong Li, and Yong-Zhong Song. Two new simple multi-symplectic schemes for the nonlinear schrödinger equation. *Chin. Phys. Lett.*, 25(5):1538, 2008.

[7] János Pipek and Imre Varga. Universal classification scheme for the spatial-localization properties of one-particle states in finite, d-dimensional systems. *Physical Review A*, 46(6):3148–3163, 1992.

[8] S. Flach. Spreading of waves in nonlinear disordered media. *Chemical Physics*, 375:548, 2010).